

**THE INFLUENCE OF QUARKS AND GLUONS JETS  
COMING FROM PRIMORDIAL BLACK HOLES  
ON THE REIONIZATION OF THE UNIVERSE.**

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**Abstract:**

In a previous work, I discussed the effect of the primordial black holes (PBHs) quantum evaporation on the reionization of the Universe at small redshifts ( $z \leq 60$ ): in principle, the photons emitted during the evaporation of such objects could drive a new ionization for the Universe after the recombination epoch ( $z \sim 1200$ ); this reionization process should happen during the last stages of the PBHs life, when they totally evaporate and emit a lot of massive and non massive particles. The critical mass of a black hole whose lifetime is equal to the present age of the Universe is  $\sim 4.4 \times 10^{14} h^{-0.3} g$ : thus, PBHs having a mass  $M \sim 10^{14} g$  are the ideal candidates to induce a reionization at small redshifts.

While in my previous study I considered an exact blackbody photon emission spectrum, here I will adopt a more realistic one, taking into account the quark and gluons jets emission through the contribution of a known fragmentation function. When the BH temperature rises above the QCD confinement scale  $\Lambda_{QCD}$ , one should expect an important contribution from quarks and gluons emission in the form of jets. In this paper I also improved my analysis by considering without any approximation the cooling effects in the plasma temperature evolution; as a result, I obtained a satisfactory “late and sudden” reionization process, characterised by a very well controlled rise of the plasma temperature: the plasma heating is not so high to induce a strong distortion of the CBR spectrum, in agreement with the recent FIRAS upper limit on the comptonization parameter,  $y_c < 2.5 \times 10^{-5}$ .

## 1. INTRODUCTION

The possibility that the Universe has been reionized after the recombination is strongly suggested by many experimental evidences as, for instance, the Gunn Peterson test [1]: the absence of the Lyman- $\alpha$  absorption line in the spectrum of high redshift quasars and the unexpected, low density of neutral hydrogen ( $n_{HI} < 10^{-11} \text{ cm}^{-3}$ ) in the intergalactic medium (IGM) suggest this hypothesis.

The causes of such a reionization for the Universe are unclear: the collisional ionization of the IGM by cosmic rays [2] or by far-UV photons produced by quasars [3] have been proposed as a possible source but, in general, a complete theory describing this phenomenon is not yet available.

An useful classification of the theoretical models presently known can be found in Ref. [4], where one distinguishes "late and sudden" (LS) models (where typically the reionization happens at  $z_R < 60$  and it is very fast) and "early and gradual" (EG) models, including, for instance, those mechanisms that assume as a ionizing source the UV radiation from decaying, massive neutrinos [5].

In Ref. [6] I discussed a possible LS reionization mechanism based on the quantum evaporation of primordial black holes: as Hawking proved [7], the evolution of a black hole involves a continuous loss of mass, a phenomenon ending with the complete evaporation of such an object. This is a typical quantum effect: in fact, in a classical sense, a black hole can only absorb particles but the quantum mechanical handling of the matter fields in a curved space-time involves some ambiguities in the decomposition of the field operators into positive and negative frequency components. Thus, the annihilation and creation operators finally work on a vacuum state that is different in regions of the space-time having a different curvature [7] and, as a consequence, the creation of particles at the expense of the black hole mass becomes possible.

As I proved in Ref. [6], the photons emitted during the evaporation process may induce the reionization of the Universe; however, in order that such a mechanism works, one should be careful the emitted photons do not heat in an excessive way the background plasma. Such a heating should contradict the FIRAS upper limit on the comptonization parameter,  $y_c < 2.5 \times 10^{-5}$  [8], a value that definitely rules out the possibility of a relevant distortion of the CBR blackbody spectrum via the Sunyaev Zel'dovich effect [9]. The proportionality of  $y_c$  to the plasma temperature clearly excludes the possibility that some processes in the past might have strongly heated the background plasma: if this is the case, very effective cooling mechanisms should have operated to cut off the temperature.

Due to some numerical problems, in Ref. [6] I solved in an approximate way the coupled differential equations system giving the evolution of the ionization degree  $x$  and of the plasma temperature  $T_e$ : thus the collisional and excitation cooling terms were underestimated.

In the following sections, I will present an improved calculation that solves the equation system taking into account in an exact way all the cooling terms; moreover, I will consider a modified, non blackbody photon emission spectrum which contains also the effect of quantum jets produced by PBHs at high temperatures ( $T > \Lambda_{QCD}$ ): the importance of such a jets emission has been put in evidence in Ref. [10] and may have some influence on the reionization process.

As a consequence of these improvements, I obtained a satisfactory evolution of both the ionization degree  $x$  and the plasma temperature  $T_e$ : in particular, the plasma heating is less relevant than in my previous study. The modification of the emission spectrum has the consequence that a lower density of primordial black holes is necessary to produce a significant reionization, because in this case the photons coming from the secondary decays of hadrons add to the ones produced by the direct evaporation; a rough estimate of the present density parameter  $\Omega_{PBH}$  inferred from this calculation is also given.

Future improvements are still possible: for instance, one may investigate how the production of massive particles, in particular electrons, might modify the background plasma and possibly influence the Inverse Compton scattering processes. Note however that the emission of charged particles represents a problem not immediately solvable because, in this case, one should also study the charge evolution of the black hole: from the theory, we know that the loss of charge and angular momentum of a charged, rotating black hole is very fast [11] but, as pointed out in Ref. [12], large, electrically charged BHs have a very interesting evolution towards the extreme Reissner-Nordström limit. During this evolution, very peculiar thermodynamic properties appear and various phase transitions may income during the evaporation process. The consequences of this particular evolution are a lifetime many order of magnitude larger than the one

of a common Schwarzschild black hole and an emission spectrum explicitly containing the charge evolution [13]: probably, in these conditions, my analysis of the reionization should be radically modified and the conclusions I drew for the uncharged case cannot be simply extended without a deep examination of the processes involved in the BHs discharge.

The consideration of the jets emission represents the first step toward a deeper understanding of the problem: in this paper, I will compare the characteristics of the reionization process both in the case of an exact Hawking spectrum and in the one when a jet fragmentation function is introduced in order to consider also the secondary photon emission from the hadron decays.

This paper is structured as follows: in Sec. 2, I will discuss the general properties of primordial black holes, in particular the characteristic of their quantum evaporation with and without jets emission; in Sec. 3, I will recall the main formulas and results of my previous work, in particular discussing the differential equations system for the variables  $x$  and  $T_e$ ; in Sec. 4 I will present an improved numerical solution method that uses the new photon spectrum and does not involve any approximations for the cooling terms; in such a way, a noteworthy improvement in the plasma temperature behaviour is obtained.

An estimate of the density of PBHs one needs in order to have a significant reionization for the Universe is also given: it corresponds to a present density parameter  $\Omega_{PBH}$  ranging from  $1.12 \times 10^{-12}$  to  $1.65 \times 10^{-8}$ , depending on the PBHs formation time one assumes.

Finally, in sec. 5 I will discuss the new results in comparison with the ones obtained in Ref. [6]; the conclusion is that the proposed model of a reionization induced by quantum evaporation of PBHs seems to work without producing an excessive plasma heating and seems to be effective in the range of reionization redshifts [15, 60] here considered. Further extensions to higher  $z_R$  values are also possible.

## 2. THE PHYSICS OF THE PRIMORDIAL BLACK HOLES: QUANTUM EVAPORATION AND PHOTON EMISSION SPECTRUM.

The fundamental work of Hawking about black holes physics [7] puts in evidence many interesting properties for such objects, in particular their peculiar connections with the general thermodynamics.

The creation of a black hole becomes possible when a mass contracts to a size less than its gravitational radius: typically, this is the case for large mass stars at the end of their evolution; however, for objects having smaller masses, a compression to huge densities is necessary in order to create a black hole; in the same time, the large pressure forces counteracting the compression should also be overcome.

For this reason, the formation of black holes having a small mass ( $M \ll M_\odot$ ) is very improbable in the contemporary Universe: however, Hawking's quantum evaporation phenomena are just important for small-mass black holes, the blackbody temperature of the emission being:

$$kT = \frac{\hbar c^3}{8\pi GM} \sim 1.06 \left[ \frac{M}{10^{13} g} \right]^{-1} GeV. \quad (2.1)$$

Therefore, our interest is mainly fixed upon the so-called *primordial* black holes (PBHs): at the early stages of the cosmological expansion, the density of the matter was really huge, thus enabling the formation of small-mass BHs.

As stressed in Ref. [14], small perturbations in a homogeneous, isotropic, hot Universe would not be able to produce relevant inhomogeneities without the contemporary presence of large fluctuations in the gravitational field; the possibility of the creation of a black hole just appears when the quantity  $l = ct$  ( $t$  is the time elapsed since the Big Bang) grows to a value in the order of the metric perturbation size and its resulting mass will be equal to the mass contained in a volume  $l^3$  at the time  $t$  [14]. The rôle of primordial density perturbations in the PBHs formation processes is discussed in a more extensive way in [15]; in particular, a study of the effect of the adiabatic inflaton quantum fluctuations in chaotic models can be found in [16]; other possible formation mechanisms might be a cosmological phase transition involving bubbles collision [17], a softening of the Universe equation of state [18] or a collapse of cosmic strings [19].

In this work, my interest is mainly concentrated on the evaporation of PBHs that form during the first stages of the expansion of the Universe: a choice of the possible birth times will be done in the next sections in connection with the calculation of the PBHs initial density we need in order to have an appreciable reionization.

The kind of the emitted particles obviously depends on the blackbody temperature of the PBHs: following eq. (2.1), the mass loss during the quantum evaporation of a BH makes it hotter and hotter, thus enabling it to produce more and more massive particles. Typically, BHs having a mass larger than  $10^{17} \text{ g}$  emit massless particles only, like photons, neutrinos and, may be, gravitons; electron production should be expected by BHs whose mass is in a range  $10^{15} \text{ g} < M < 10^{17} \text{ g}$  and muon production for a range  $10^{14} \text{ g} < M < 10^{15} \text{ g}$ .

Here I will consider PBHs that survive till the post-recombination epoch, thus having a mass  $M \sim O(10^{14}) \text{ g}$ , near to the critical mass,  $M_c \sim 4.4 \times 10^{14} h^{-0.3} \text{ g}$ . These PBHs should reionize the Universe at a redshift  $z_R$  that corresponds to the time of their complete evaporation.

The initial mass  $M_i$  is connected to the lifetime of a PBH by the following formula [20]:

$$t_{evap} \sim 1.19 \times 10^3 \frac{G^2 M_i^3}{\hbar c^4 f(M_i)} \sim 6.24 \times 10^{-27} f(M_i)^{-1} M_i^3 \text{ sec}, \quad (2.2)$$

where the function  $f(M)$  contains the contributions of the different species of particles and it is normalized to the unit for very massive ( $M \geq 10^{17} \text{ g}$ ) BHs, emitting massless particles only; a suitable way to express these contributions is given by the following formula [20]:

$$\begin{aligned} f(M) = & 1.569 + 0.569 \left[ \exp \left[ \frac{-M}{4.53 \cdot 10^{14}} \right]_\mu + 6 \exp \left[ \frac{-M}{1.60 \cdot 10^{14}} \right]_{u,d} + \right. \\ & + 3 \exp \left[ \frac{-M}{9.60 \cdot 10^{13}} \right]_s + 3 \exp \left[ \frac{-M}{2.56 \cdot 10^{13}} \right]_c + \exp \left[ \frac{-M}{2.68 \cdot 10^{13}} \right]_\tau + \\ & + 3 \exp \left[ \frac{-M}{9.07 \cdot 10^{12}} \right]_b + 3 \exp \left[ \frac{-M}{0.48 \cdot 10^{12}} \right]_t \Big] + \\ & \left. + 0.963 \left[ \exp \left[ \frac{-M}{1.10 \cdot 10^{14}} \right]_{gluons} \right] \right]. \end{aligned} \quad (2.3)$$

In eq. (2.3) the first addendum in the right-hand side expresses the contribution of electrons, positrons, photons and neutrinos; heavier particles contributions are considered in the remaining terms, following their relative importance; the factor 3 takes into account the presence of the color charge for quarks and the denominators in the exponential terms are defined as the product  $\beta_{sj} M_j$ , where  $M_j$  is the mass of a black hole whose temperature is equal to the rest mass  $\mu_j$  of the  $j$  species and  $\beta_{sj}$  is a spin-dependent factor defined [20] in such a way the energy of a BH having  $M = \beta_{sj} M_j$  has a peak at  $\mu_j$ .

Eq. (2.3) is fundamental in the determination of the mass evolution of a PBH that evaporates: in fact, in Refs. [10], [20], Carr, Mac Gibbon and Webber show that the quarks and gluons emission processes should not be neglected at the stages when the BH mass falls below  $10^{14} \text{ g}$  and the temperature  $T$  becomes larger than the confinement scale  $\Lambda_{QCD}$ : at this time one should expect a copious production of jets and this is the case we are interested in, due to the particular range of initial masses chosen for the PBHs.

Through the function  $f(M)$ , eq. (2.3), we can take into account the quarks/gluons emission processes: the mass evolution is given by:

$$\frac{dM}{dt} = - \sum_j \frac{1}{2\pi\hbar} \int \Gamma_j \left[ \exp \left[ \frac{8\pi GQM}{\hbar c^3} \right] - (-1)^{2s_j} \right]^{-1} \times \frac{Q dQ}{c^2}; \quad (2.4)$$

eq. (2.4) means that the emission of a parent particle  $j$  with total energy  $Q$  decreases the BH mass by  $Q/c^2$ ; here  $\Gamma_j$  is the absorption probability for the  $j$  particle having a spin  $s_j$  [21] and a sum on all the emitted species has been performed [20]. After the integration over the energy  $Q$ , eq. (2.4) can be rewritten as [20]:

$$\frac{dM}{dt} = -5.34 \times 10^{25} f(M) M^{-2} \text{ g sec}^{-1} \quad (2.5)$$

Now, the Hawking emission rate of particles having an energy in the range  $(E, E + dE)$  from a black hole having an angular velocity  $\omega$ , an electric potential  $\phi$  and a surface gravity  $\kappa$  is [7]:

$$\frac{dN}{dt} = \frac{\Gamma}{2\pi\hbar} \left[ \exp \left( \frac{E - n\hbar\omega - e\phi}{\hbar\kappa/2\pi c} \right) \pm 1 \right]^{-1}, \quad (2.6)$$

where the signs  $\pm$  respectively refer to fermions and bosons and  $\Gamma$  is the absorption probability of the emitted species: for photons, it reads as [21]:

$$\Gamma_{s=1} = \frac{4A}{9\pi} \left( \frac{M}{M_{PL}} \right)^2 \left( \frac{\omega}{\omega_{PL}} \right)^4; \quad (2.7)$$

here  $A$  is the surface area of the BH and the Planck mass and energy have been introduced in order to work with dimensionless quantities as in Ref. [21].

In most cases, one assumes that the charge and the angular momentum of a black hole are negligible because, as Page proved [13], their loss through quantum evaporation happens on a time scale shorter than the one characterizing the mass loss; this assumption is certainly reasonable but one should also note that in some way it may be relevant to determine the influence of the black hole charge evolution (in particular studying the discharge process) in connection with the modification of the electron background; thus, one should rather follow the approach suggested in ref. [12].

In Ref. [6] I studied the reionization of the Universe induced by quantum evaporation of PBHs by using a photon emission spectrum as the one of eq. (2.6), i.e. an exact blackbody spectrum; this assumption enabled me just to test if the proposed model of reionization might work at least under very general conditions. After this satisfactory test, I will now improve my previous calculation by studying the effect of the presence of a jet fragmentation function contribution in the emission spectrum: then, eq. (2.6) should be rewritten as [22]:

$$\frac{dN_x}{dt dE} = \sum_j \int_0^{+\infty} \frac{\Gamma_j(Q, T)}{2\pi\hbar} \left( \exp \frac{Q}{T} \pm 1 \right)^{-1} \frac{dg_{jx}(Q, E)}{dE} dQ; \quad (2.8)$$

here  $x$  and  $j$  respectively refer to the final and the directly emitted particles and the last factor, containing the fragmentation function  $g_{jx}$ , expresses the number of particles with energy in the range  $(E, E + dE)$  coming from a jet having an energy equal to  $Q$ : namely, it reads as [22]:

$$\frac{dg_{jx}(Q, E)}{dE} = \frac{1}{E} \left( 1 - \frac{E}{Q} \right)^{2m-1} \theta(E - km_h c^2), \quad (2.9)$$

where  $m_h$  is the hadron mass,  $k$  is a constant  $O(1)$  and  $m$  is an index equal to 1 for mesons and 2 for baryons.

After determining the dominant contribution to the integral over  $Q$  and summing over the final states, one can approximate eq. (2.8) by the following formulas [22]:

$$\frac{dN}{dt dE} \sim E^2 \exp \left( \frac{-E}{T} \right) \quad \text{for } E \gg T \quad Q \sim E, \quad (2.10a)$$

$$\frac{dN}{dt dE} \sim E^{-1} \quad \text{for } T \sim E \gg m_h \quad Q \sim T, \quad (2.10b)$$

$$\frac{dN}{dt dE} \sim \frac{dg}{dE} \quad \text{for } E \sim m_h \ll T \quad Q \sim m_h, \quad (2.10c)$$

where the different expressions refer to the specified  $Q$ -value that dominates. A more exhaustive discussion of the characteristics of the spectrum given by eqs. (2.10a), (2.10b) and (2.10c) can be found in Ref. [22].

In the next Section, I will examine the problem of the reionization of the Universe by considering as a source of the ionizing photons the quantum evaporation of PBHs in presence of quarks and gluons jets.

### 3. THE TIME EVOLUTION OF THE IONIZATION DEGREE

## AND PLASMA TEMPERATURE FOR A REIONIZED UNIVERSE.

As I discussed in Ref. [6], the basic equations that control the ionization degree  $x$  and the plasma temperature  $T_e$  evolution with the time are [23]:

$$\frac{dx}{dt} = t_{pi}^{-1} + t_{coll}^{-1} - t_r^{-1}, \quad (3.1)$$

and

$$\frac{dT_e}{dt} = -2 \frac{\dot{R}}{R} T_e - \frac{T_e}{(1+x)} \frac{dx}{dt} + \frac{2}{3(1+x)} (\Gamma - \Lambda), \quad (3.2)$$

where  $t_{pi}^{-1}$ ,  $t_{coll}^{-1}$ ,  $t_r^{-1}$  are, respectively, the photoionization, the collisional and the recombination rates, given by [23]:

$$t_{pi}^{-1} = \frac{8\pi m_e \alpha^5}{3\sqrt{3}} (1-x) \frac{1}{\pi^2} \int_{\omega_{min}}^{\omega_{max}} d\omega \frac{n_\gamma}{\omega^3}; \quad (3.3)$$

$$t_r^{-1} \sim 10^{-13} \text{ sec}^{-1} \left(\frac{\Delta}{T_e}\right)^{1/2} x^2 \left(\frac{\Omega_b h^2}{0.025}\right) \left(\frac{1+z}{200}\right)^3 \quad (\Delta = 1 \text{ Ry}); \quad (3.4)$$

$$t_{coll}^{-1} = 6 \times 10^{-8} \text{ sec}^{-1} \left(\frac{T_e}{\Delta}\right)^{1/2} e^{-\Delta/T_e} x (1-x) \left(\frac{\Omega_b h^2}{0.025}\right) \left(\frac{1+z}{200}\right)^3. \quad (3.5)$$

Eq. (3.1) expresses the combined effects of the photoionization and the recombination and the influence of the collisional interactions putting the neutral hydrogen in an excited or ionized state. In eq. (3.2), the heating  $\Gamma$  mainly comes from the photoionization process and reads as

$$\Gamma = \Gamma_{pi} = \frac{8\pi m_e \alpha^5}{3\sqrt{3}} (1-x) \frac{1}{\pi^2} \int_{\omega_{min}}^{\omega_{max}} d\omega \frac{(\omega - \Delta)}{\omega^3} n_\gamma; \quad (3.6)$$

on the contrary, the cooling  $\Lambda$  takes into account a lot of contributions having a relevant importance in order to limit the rise of the plasma temperature induced by the quantum evaporation of PBHs. These contributions are due to the recombination, collisional and excitation processes, Compton scattering and to the adiabatic cooling due to the expansion of the Universe. The total cooling reads as

$$\Lambda = \Lambda_r + \Lambda_{exc} + \Lambda_{coll} + \Lambda_{Compt} + \Lambda_{exp}, \quad (3.7)$$

where, respectively:

$$\Lambda_r = 2 \times 10^{-21} \text{ GeV sec}^{-1} \left(\frac{T_e}{\Delta}\right)^{1/2} x^2 \left(\frac{\Omega_b h^2}{0.025}\right) \left(\frac{1+z}{200}\right)^3; \quad (3.8)$$

$$\Lambda_{exc} = 10^{-15} \text{ GeV sec}^{-1} e^{-3\Delta/4T_e} \left(\frac{T_e}{\Delta}\right)^{1/2} x (1-x) \left(\frac{\Omega_b h^2}{0.025}\right) \left(\frac{1+z}{200}\right)^3; \quad (3.9)$$

$$\Lambda_{coll} = 8 \times 10^{-16} \text{ GeV sec}^{-1} \left(\frac{T_e}{\Delta}\right)^{1/2} e^{-\Delta/T_e} x (1-x) \left(\frac{\Omega_b h^2}{0.025}\right) \left(\frac{1+z}{200}\right)^3; \quad (3.10)$$

$$\Lambda_{Compt} = 10^{-19} \text{ GeV sec}^{-1} x \left(\frac{T_e - T_{CMB}}{\Delta}\right) \left(\frac{1+z}{200}\right)^4; \quad (3.11)$$

$$\Lambda_{exp} = 2 \times 10^{-22} \text{ GeV sec}^{-1} \left(\frac{T_e}{\Delta}\right) (1+x) \left(\frac{1+z}{200}\right)^{1.5}. \quad (3.12)$$

( $T_{CMB}$  is the Cosmic Microwave Background temperature). Both the equations (3.3) and (3.6) explicitly contain the photon number density  $n_\gamma$ : the time evolution of this variable is quite difficult to establish

because it depends on many processes not simply correlated; the best way to describe this evolution is through a differential equation that reads as follows [6]:

$$\begin{aligned} \frac{\partial n_\gamma(\omega, t)}{\partial t} + \frac{\dot{R}}{R} \frac{\partial n_\gamma(\omega, t)}{\partial \omega} \omega_{NOR} - 2 \frac{\dot{R}}{R} \frac{n_\gamma(\omega, t)}{\omega} \omega_{NOR} = \\ = \left( \frac{dn_{\gamma R}}{dt} - \frac{dn_{\gamma PI}}{dt} \right) + \frac{2}{\omega} \frac{d\omega}{dt} [n_{\gamma PI} - n_{\gamma R}] + \frac{\partial n_{\gamma PR}}{\partial \omega \partial t} \omega_{NOR}; \end{aligned} \quad (3.13)$$

here  $n_{\gamma PI}$ ,  $n_{\gamma R}$  are the photon number densities respectively involved in the processes of photoionization and recombination and  $\omega_{NOR}$  is a normalization factor, equal to  $10^{-6} \text{ GeV}$ : this normalization assures one is working with dimensionless quantities in the numerical routine that solves the differential equation system. The last term in eq. (3.13) is the contribution of the photon source, in our case a number  $n_{PBH}$  of primordial black holes that evaporate.

From the basic work of Peebles [24], the first term on the right-hand side of eq. (3.14) has the following form:

$$\left( \frac{dn_{\gamma R}}{dt} - \frac{dn_{\gamma PI}}{dt} \right) = - \frac{dn_e}{dt} = C \left( \alpha_e n_e^2 - \beta_e n_{1s} e^{-\omega_\alpha/T} \right) - \frac{n_e}{n} \frac{dn}{dt}. \quad (3.14)$$

Explicitly,  $\alpha_e n_e^2$  is the rate of recombination to excited states, ignoring the recombination direct to the ground state,  $\alpha_e = \langle \sigma \nu \rangle$  is the recombination coefficient and  $\beta_e n_{1s} e^{-\omega_\alpha/T}$  expresses the rate of photoionization;  $\beta_e$  is a constant proportional to  $\alpha_e$ ,  $n_{1s}$  is the population of the  $1s$  state of the hydrogen atom,  $\omega_\alpha$  is the energy corresponding to the transition  $L_\alpha$ ,  $T$  the radiation temperature and  $n$  is the nucleon number density. Finally,  $C$  is a reduction factor which takes into account the effect of the  $L\alpha$  resonance photons. All the numerical values of these quantities can be found in Ref. [25]; really, we do not mind this contribution because it proves to be negligible with respect to the difference  $n_{\gamma PI} - n_{\gamma R}$ .

Now, as in Ref. [6], I put

$$n_\gamma = n_{\gamma PR} - n_{\gamma PI} + n_{\gamma R}, \quad (3.15)$$

and with the same substitution performed in [6] I obtain:

$$\begin{aligned} \frac{\partial n_\gamma(\omega, t)}{\partial t} + 2 \frac{n_\gamma}{\omega} \left[ - \frac{\dot{R}}{R} \omega_{NOR} + \frac{d\omega}{dt} \right] + \frac{\dot{R}}{R} \frac{\partial n_\gamma(\omega, t)}{\partial \omega} \omega_{NOR} = \\ = C \left( \alpha_e x^2 n^2 - \beta_e n_{1s} e^{-\omega_\alpha/T} \right) - x \frac{dn}{dt} + \frac{\partial n_{\gamma PR}}{\partial \omega \partial t} \left( \frac{2}{\omega} \frac{d\omega}{dt} + \omega_{NOR} \right). \end{aligned} \quad (3.16)$$

At this point my present analysis of the problem differs from the approach I followed in Ref. [6]: in fact, here I will take as a source term in eq. (3.13) the photon emission spectrum corrected by using a jet fragmentation function (eqs. (2.8), (2.10a), (2.10b), (2.10c)) while previously I used an exact Hawking photon spectrum, eq. (2.6).

#### 4. THE NUMERICAL SOLUTION OF THE DIFFERENTIAL EQUATION SYSTEM.

I will start the present analysis of the reionization mechanism by evaluating the total photon number density coming from the quantum evaporation of PBHs: eqs. (2.10a), (2.10b) and (2.10c) give the emission rate for various values of the energy characterizing the emitted particles.

As I stressed in Ref. [6], I would take into account only the contribution of those photons that may really reionize the Universe, i.e., I want to select the components of the emission spectrum really effective by comparing the time scale characterizing the photon interactions with the expansion time for the Universe. The following condition should be satisfied:

$$t_{int} < t_{exp}. \quad (4.1)$$

As a consequence of the condition (4.1), the latest time at which the photon interactions may be relevant is [10]:

$$t_{free} = (n_X \sigma)^{-1}. \quad (4.2)$$

( $n_X$  = number density of the background particles,  $\sigma$  = interaction cross-section).

Among the possible interaction processes, I want to consider the ionization and the recombination: the range of energy where these processes are dominating is typically  $E_\gamma \leq 14 \text{ KeV}$  but, indeed, the maximum allowed energy for photons having a rôle in the reionization of the Universe is further constricted by the condition (4.1); in the pregalactic, matter-dominated era, a photon is affected by ionization losses only before a redshift [10]:

$$1 + z_{free} \sim 1.0 \times 10^{19} h^{-2} \left( \frac{0.2}{\Omega_p} \right) E^{3.5}, \quad (4.3)$$

where  $\Omega_p \sim 0.2$  is the present density parameter for protons,  $h = 0.5$  and the energy is expressed in  $\text{GeV}$ .

Eq. (4.3) can be inversely used in order to determine  $E_{max}$  at a particular value of  $z$ , corresponding to the reionization redshift  $z_R$ : in tab. 1 I listed the values I obtained, typically smaller than  $8 \text{ KeV}$  for a late reionization with  $z_R < 60$ .

Coming back to the eqs. (2.10a), (2.10b) and (2.10c), due to the upper limit we have for the photon energy, the form of the spectrum is the one expressed by eq. (2.10c), holding in the case  $E \ll T$ .

Taking the jet fragmentation function as in eq. (2.9) and separating the mesons and baryons contributions, I can write:

$$\frac{\partial n_\gamma}{\partial \omega \partial t}|_{Tot} = \frac{\partial n_\gamma}{\partial \omega \partial t}|_{Mes} + \frac{\partial n_\gamma}{\partial \omega \partial t}|_{Bar}, \quad (4.4)$$

(for convenience, in the following discussion the energy of the final photons will be called  $\omega$ ).

For  $Q = m_{hadr} \sim 300 \text{ MeV}$ , eq. (4.4) simply reads:

$$\frac{\partial n_\gamma}{\partial \omega \partial t}|_{tot} = \frac{1}{\omega} \left( 1 - \frac{\omega}{Q} \right) + \frac{1}{\omega} \left( 1 - \frac{\omega}{Q} \right)^3, \quad (4.5)$$

the dominating contribution being the mesonic one.

Turning now to the solution of the photon density equation, it is useful to adopt dimensionless rescaled variables,  $\tilde{\omega} = \omega / \omega_{NOR}$  with  $\omega_{NOR} = 10^{-6} \text{ GeV}$ ,  $\tilde{M} = M / 10^{14} \text{ g}$  and  $\tilde{t} = t / 10^{15} \text{ sec}$  and to write eq. (3.16) in the form

$$\frac{\partial n_\gamma}{\partial \tilde{t}} + \frac{2}{3\tilde{t}} \frac{\partial n_\gamma}{\partial \tilde{\omega}} = \frac{4}{3\tilde{t}} \frac{n_\gamma}{\tilde{\omega}} - 2 \frac{n_\gamma}{\tilde{\omega}} \frac{d\tilde{\omega}}{d\tilde{t}} + \frac{\partial n_{\gamma PR}}{\partial \tilde{t} \partial \tilde{\omega}} \left( 1 + \frac{2 \times 10^{-9}}{\tilde{\omega}} \frac{d\tilde{\omega}}{d\tilde{t}} \right). \quad (4.6)$$

In eq. (4.6) the constant  $2 \times 10^{-9}$  is a numerical factor coming from the transformation to the new rescaled variables and all the negligible terms have been skipped away.

From the explicit calculation of eq. (4.5), one has:

$$\frac{\partial n_{\gamma PR}}{\partial \tilde{t} \partial \tilde{\omega}} \sim \frac{2 \times 10^{15}}{\tilde{\omega}}. \quad (4.7)$$

Now, the general solution of eq. (4.6) can be found in an analytical way by using the method discussed in Ref. [6]: here, I would like just to recall that the general solution of a first order, linear partial differential equation that reads as

$$Pp + Qq = R, \quad (4.8)$$

( $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$  and  $P$ ,  $Q$ ,  $R$  are any function of the general variables  $x$ ,  $y$ ,  $z$ ) is a function  $F(u, v)$  that also solves [26] the equations

$$\left| \frac{dx}{P} \right| = \left| \frac{dy}{Q} \right| = \left| \frac{dz}{R} \right|; \quad (4.9)$$

here  $u(x, y, z) = c_1$ ,  $v(x, y, z) = c_2$  are implicit functions and the constants  $c_1$ ,  $c_2$  should be fixed by imposing suitable boundary conditions to the particular problem one is studying.

After the identification of the first variable,  $x$ , with the time  $\tilde{t}$ , the second one,  $y$ , with the energy  $\tilde{\omega}$  and finally  $z$  with the photon number density  $n_\gamma$ , I obtain:

$$P = 1 \quad Q = \frac{2}{3\tilde{t}}, \quad (4.10a)$$

$$R = \frac{8}{3\tilde{t}} \frac{n_\gamma}{\tilde{\omega}} + \frac{2 \times 10^{15}}{\tilde{\omega}}; \quad (4.10b)$$

the main difference with the expression found in ref. [6] consists in the modified source photon term. Contrarily to my previous analysis, here I do not examine the very particular case when one has equilibrium between photoionization and recombination.

The left-hand side of the equality (4.9) gives:

$$\frac{d\tilde{\omega}}{d\tilde{t}} = -\frac{2}{3\tilde{t}}, \quad (4.11)$$

as in Ref. [6], while for the photon density  $n_\gamma$  I calculate

$$\frac{dn_\gamma}{d\tilde{\omega}} = \frac{3\tilde{t}}{2} R = \frac{1}{\tilde{\omega}} \left[ 4n_\gamma + \frac{3\tilde{t}}{2} a \right], \quad (4.12)$$

where  $a = 2 \times 10^{15}$ . After solving the linear equation (4.12) one finds:

$$n_\gamma(\tilde{\omega}) = -\frac{3}{8}a\tilde{t} + c_2\tilde{\omega}^4; \quad (4.13)$$

the integration constant  $c_2$  is fixed by imposing that the final number of emitted photon is zero for  $t = t_0$ :

$$c = \frac{3}{8}a\frac{\tilde{t}_0}{\tilde{\omega}_0^4}, \quad (4.14)$$

where  $\tilde{t}_0 = t_0/10^{15}$  sec and a good choice for  $\tilde{\omega}_0$  should be done in such a way one has a positive photon density at all times; from such a constraint, one obtains  $\tilde{\omega}_0 \sim \tilde{\omega}_{max}/12$ ; moreover, a lower limit on the photons energy comes from the request that the emission process is not less than the recombination losses: this minimum value is equal to 0.20 KeV while, as I discussed in the previous sections, the maximum energy depends on the reionization redshift  $z_R$ .

I adopted a numerical Merson routine to solve eqs. (3.1), (3.2): note however that, although Merson is a very powerful method, it fails if the photon number density evolution as expressed by eq. (4.14) is directly inserted in eqs. (3.3), (3.6) for  $t_{PI}^{-1}$  and  $\Gamma_{PI}$ . Really, Merson cannot control the contemporary evolution of two variables like the time and the photon density in the same equation; a solution is only possible by making some simplifying assumptions, namely:

- a) the collision contribution  $t_{coll}^{-1}$  in eq. (3.1) is neglected with respect to the photoionization one: this assumption is certainly acceptable if we suppose that the quantum evaporation phenomena is the most effective contribution to the photon number density. In this way, Merson should not control terms having coefficients different by many order of magnitude.
- b) In eqs. (3.3), (3.6) I took  $\omega_{min} = 0.20$  KeV, i.e. the minimum of the energy previously obtained; then I put:

$$A_1(\tilde{t}) = \int_{\tilde{\omega}_{min}}^{\tilde{\omega}_{max}} d\tilde{\omega} \frac{n_\gamma(\tilde{\omega})}{\tilde{\omega}^3}; \quad (4.15a)$$

$$B_1(\tilde{t}) = \omega_{NOR} \int_{\tilde{\omega}_{min}}^{\tilde{\omega}_{max}} d\tilde{\omega} (\tilde{\omega} - \Delta) \frac{n_\gamma(\tilde{\omega})}{\tilde{\omega}^3}. \quad (4.15b)$$

Now, I insert in eqs. (3.1) and (3.2) the time-averaged values  $\overline{A_1}$ ,  $\overline{B_1}$  that I quoted in tab. 2 for various reionization redshifts: in such a way, the Merson routine must solve a constant coefficients equation system and no approximations are necessary in the handling of the cooling terms of eq. (3.2).

On the contrary, in ref. [6] I tried to maintain the time dependence in the functions  $A_1$ ,  $B_1$  but I was obliged to insert some cooling terms in an approximate form; the present choice seems most suitable also because the use of an average number of photons in eq. (3.1), (3.2) probably does not represent an extreme approximation, due to our poor experimental knowledge of the primeval PBH density parameter  $\Omega_{PBH}$ ; further improvements in the mathematical solution of the system are under study.

As a result of this solution method, I am able to control the rise in the plasma temperature  $T_e$  while previously the model involved a too large effect of plasma heating. The effectiveness of the cooling mechanism can disclose if the suppression due to the negative exponential terms in eqs. (3.9), (3.10), i.e. to the excitation / collisional terms, is avoided by choosing to start with a high plasma temperature  $T_e$  corresponding to an instantaneous heating.

Remembering this assumptions, the couple of differential equations is written as follows:

$$\frac{dx}{dt} = C_1(z_R) n_{PBH} (1-x) - 77.45 \frac{x^2}{\tilde{t}^2} \tilde{T}_e^{-1/2}; \quad (4.16a)$$

$$\begin{aligned} \frac{d\tilde{T}_e}{dt} = & -1.33 \frac{\tilde{T}_e}{\tilde{t}} - \frac{\tilde{T}_e}{(1+x)} \frac{dx}{dt} + \frac{0.67}{(1+x)} \cdot \left[ C_2(z_R) n_{PBH} (1-x) - \right. \\ & - 113.40 \frac{x^2}{\tilde{t}^2} \tilde{T}_e^{1/2} - 5.69 \times 10^7 x (1-x) \frac{1}{\tilde{t}^2} \left(1 - \frac{1020}{\tilde{T}_e}\right) \tilde{T}_e^{1/2} - \\ & - 4.54 \times 10^7 x (1-x) \frac{1}{\tilde{t}^2} \left(1 - \frac{1360}{\tilde{T}_e}\right) \tilde{T}_e^{1/2} - 44.10 x \left(\tilde{T}_e - \frac{1.27}{\tilde{t}^{2/3}}\right) \frac{1}{\tilde{t}^{8/3}} - \\ & \left. - 2.13 (1+x) \frac{\tilde{T}_e}{\tilde{t}} \right]; \end{aligned} \quad (4.16b)$$

here  $\tilde{T}_e = T_e/10^{-11} \text{ GeV}$ .

In eq. (4.16a) I neglected the collisional term  $t_{coll}^{-1}$  (as I explained above); thus, the addends in eq. (4.16a) respectively correspond to  $t_{pi}^{-1}$ ,  $t_r^{-1}$  and the ones in brackets in eq. (4.16b) to  $\Gamma_{pi}$ ,  $\Lambda_r$ ,  $\Lambda_{exc}$ ,  $\Lambda_{coll}$ ,  $\Lambda_{Compt}$  and  $\Lambda_{exp}$ . Finally, the coefficients  $C_1(z_R)$ ,  $C_2(z_R)$  are given by:

$$C_1(z_R) = 7.88 \times 10^{24} \overline{A}_1(z_R), \quad (4.17a)$$

$$C_2(z_R) = 7.88 \times 10^{35} \overline{B}_1(z_R), \quad (4.17b)$$

and they are listed in tab. 3 for various  $z_R$  from 15 to 60.

In eqs. (4.16a) and (4.16b), the parameter  $n_{PBH}$  is a weight factor that represents the effective number of PBHs in the early Universe we need in order to produce an appreciable reionization effect; a suitable value for this free parameter is  $n_{PBH} = 10^{-44}$ ; an estimate in terms of an effective initial number density of PBHs corresponding to this value of  $n_{PBH}$  can be roughly obtained by considering the following relations:

$$\rho_i \sim \frac{\langle M \rangle n_{PBH}}{R^3(t_{in})}, \quad (4.18)$$

where  $\langle M \rangle \sim 10^{14} \text{ g}$  and

$$R(t_{in}) \sim R_0 (t_0/t_{in})^\alpha; \quad (4.19)$$

here  $R_0 = 1.25 \times 10^{28} \text{ cm} = 1.4 \times 10^{10} \text{ lyr}$  in a typical cosmological model [27] while for  $\alpha$  we can take the value  $-0.5$ . Then, if one assumes for the PBHs density a behaviour that approximately scales as a power with exponent  $2/3$  of the time and an initial time  $t_{in} \sim 10^{-70} \div 10^{-75} \text{ sec}$ , the density parameter  $\Omega_{PBH}$  at the present time is:

$$\Omega_{PBH} = \frac{\rho_{0PBH}}{\rho_{cr}} = 1.12 \times 10^{-12} \div 1.65 \times 10^{-8}; \quad (4.20)$$

In tab. 4 I listed the results obtained by choosing different formation times for the primordial black holes: this rough estimate is purely indicative but in any case it may reproduce quite well the present experimental upper limit of the density parameter, derived from the constraints imposed by studying the CMB characteristics [10]:  $\Omega_{PBH} \leq (7.6 \pm 2.6) \times 10^{-9} h^{(-1.95 \pm 0.15)}$ .

## 5. RESULTS AND CONCLUSIONS.

In figs. 1-6 I plotted the behaviour of the ionization degree  $x$  vs  $-z$ , as results from the solution of eq. (4.16a): the six different plots refer to values of the reionization redshift  $z_R$  ranging from 15 to 60; in figs. 7-12 I plotted the evolution vs  $-z$  of the plasma temperature; for convenience, I also reported in figs. 13-18 and 19-24 the corresponding results obtained in Ref. [6] by using an exact blackbody spectrum without jets emission.

The present results should be compared with the ones obtained in [6] by a first order solution of the differential equation system; we can remark that:

- a) as in Ref. [6], the black holes-induced reionization is only partial; a quite relevant effect is obtained for an evaporation redshift corresponding to  $z_R \leq 30$ , while for higher values of  $z_R$  the process of PBHs quantum evaporation cannot produce an appreciable phenomenon of reionization; in fact, for  $z_R \geq 40$  the ionization degree is  $x \leq 0.35$ . However, with respect to the results of Ref. [6], a smaller density of PBHs is necessary to reionize the Universe because in this case, one adds to the direct photon emission the indirect one, due to the hadrons decays. Moreover, the plasma heating is limited by a powerful cooling: the overall effect is that in eq. (3.1) the  $t_{pi}^{-1}$  term is suppressed and the  $t_r^{-1}$  term enhanced.
- b) Discussing now the result obtained for the plasma temperature and looking at figs. 7-12, one can remark that a real improvement of the calculation has been obtained: the consideration of all the cooling terms without any approximations enables me to have a maximum plasma temperature of the order of the tenth of eV while previously the PBHs quantum evaporation was producing an excessive plasma heating; the irregularities in the plots are due to some numerical fluctuations attributable to the routine solving the coupled differential equations system.

These fluctuations are more evident when a shorter integration time is considered.

- c) Looking at both the results for the ionization degree and the plasma temperature, one can conclude that this PBHs-induced reionization mechanism should effectively be “late and sudden”: the possibility of an effective reionization for values  $z_R > 30$  should be discarded and thus, in this model, it is very improbable to have a significative suppression of the CMB temperature fluctuations on small angular scales as predicted by Bond and Efstathiou [28] and Vittorio and Silk [29]; in these works, one proves that an early reionization could suppress in a significant way these fluctuations, that, following the predictions of CDM models and texture scenarios [30], result too large.
- d) The PBHs formation time should be put very far in the past: considering the jet emission contribution in the photon spectrum, the best choice of the PBHs formation conditions that enables to obtain a well balanced reionization process (i.e. an high ionization degree without an excessive plasma heating) suggests a formation time in a range  $t_{form} = 10^{-70} \div 10^{-75}$  sec after the Big Bang and an initial density  $\rho_i = 10^{17} \div 10^{24} g/cm^3$  (see tab. 4), corresponding to a present density parameter  $\Omega_{PBH} \sim 1.12 \times 10^{-12} \div 1.65 \times 10^{-8}$ .

Summarizing, in this paper I studied a mechanism of reionization for the Universe induced by the quantum evaporation of primordial black holes. Differently from a previous analysis, here an emission spectrum taking into account the quark and gluons jets production has been considered.

A most accurate solution of the system of coupled differential equations, giving the ionization degree and the plasma temperature evolution with the time, is performed; as a result, the reionization of the Universe happens in an effective, even if partial, way. In the same time, the resulting plasma heating is limited by an effective cooling, due to excitation and collisional processes; that avoids to have large (experimentally unseen) distortions of the CBR blackbody spectrum via the Sunyaev- Zel'dovich effect.

As in Ref. [6], I considered here some values of the reionization redshift in the range [15, 60]: the fast rise of the ionization degree  $x$  seems to suggest that such a model of PBHs-induced reionization should be classified as “sudden and late”. In fact, the behaviour of  $x$  can be approximated by an exponential function: this result justifies the analysis of Ref. [31], where I studied the consequences of an exponential reionization of the Universe on the polarization of the Cosmic Microwave Background.

Finally, the possibility to have a total reionization for a redshift  $z_R > 60$  seems to be excluded by the results here found: in this model, no damping of the temperature fluctuations at small angular scales should be expected.

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**Tab. 1:** Maximum energy for photons affected by ionization losses before a redshift  $\tilde{z}$ .

$\tilde{z}$	$\omega_{max} (\times 10^{-6}) \text{ GeV}$
15	5.54
20	6.00
30	6.70
40	7.20
50	7.70
60	8.10

**Tab. 2:** Time averaged values of the integrated photon density  $A_1$ ,  $B_1$ .

$z_R$	$t_R$	$\overline{A_1}$	$\overline{B_1}$
15	$6.43 \times 10^{15} \text{ sec}$	$10.4 \times 10^{19}$	$3.83 \times 10^{14}$
20	$4.30 \times 10^{15} \text{ sec}$	$8.88 \times 10^{19}$	$3.54 \times 10^{14}$
30	$2.38 \times 10^{15} \text{ sec}$	$7.12 \times 10^{19}$	$3.17 \times 10^{14}$
40	$1.57 \times 10^{15} \text{ sec}$	$6.17 \times 10^{19}$	$2.96 \times 10^{14}$
50	$1.13 \times 10^{15} \text{ sec}$	$5.40 \times 10^{19}$	$2.76 \times 10^{14}$
60	$0.86 \times 10^{15} \text{ sec}$	$4.88 \times 10^{19}$	$2.63 \times 10^{14}$

**Tab. 3:** Value of the coefficients  $C_1(z_R)$ ,  $C_2(z_R)$  respectively appearing in  $t_{PI}^{-1}$  and  $\Gamma_{PI}$ .

$z_R$	$C_1$	$C_2$
15	$8.20 \times 10^{44}$	$3.02 \times 10^{50}$
20	$6.99 \times 10^{44}$	$2.79 \times 10^{50}$
30	$5.61 \times 10^{44}$	$2.50 \times 10^{50}$
40	$4.86 \times 10^{44}$	$2.33 \times 10^{50}$
50	$4.25 \times 10^{44}$	$2.18 \times 10^{50}$
60	$3.84 \times 10^{44}$	$2.07 \times 10^{50}$

**Tab. 4: Values of the density parameter  $\Omega_{PBH}$  and of the PBHs number densities for different choices of the initial black holes birth time  $t_i$  (Exp:  $\Omega_{PBH} \leq (7.6 \pm 2.6) \times 10^{-9} h^{(-1.95 \pm 0.15)}$ ).**

$t_i$ (sec)	$\rho_i$ ( $g\ cm^{-3}$ )	$\rho_0$ ( $g\ cm^{-3}$ )	$\Omega_{PBH}$
$10^{-75}$	$4.28 \times 10^{24}$	$7.74 \times 10^{-38}$	$1.65 \times 10^{-8}$
$10^{-73}$	$4.28 \times 10^{21}$	$1.66 \times 10^{-39}$	$3.53 \times 10^{-10}$
$10^{-71}$	$4.28 \times 10^{18}$	$3.59 \times 10^{-41}$	$7.64 \times 10^{-12}$
$10^{-70}$	$1.35 \times 10^{17}$	$5.26 \times 10^{-42}$	$1.12 \times 10^{-12}$

## FIGURE CAPTIONS

Fig. 1: Evolution of the ionization degree  $x$  vs  $-z$  induced by primordial black holes evaporation; the Hawking emission spectrum is corrected for the presence of quarks and gluons jets; the plot is obtained by assuming a reionization redshift  $z = 15$ .

Figs. 2-6: The same evolution of  $x$  but, respectively, for  $z = 20, z = 30, z = 40, z = 50, z = 60$ .

Fig. 7: The evolution of the plasma temperature  $T_e$  vs  $-z$ ; the reionization redshift is  $z = 15$ .

Fig. 8-12: The same evolution of  $T_e$  but, respectively, for  $z = 20, z = 30, z = 40, z = 50, z = 60$ .

Fig. 13: The evolution of the ionization degree  $x$  is calculated by using an exact blackbody emission spectrum in a first order recursive calculation (see Ref. 6) and plotted as a function of  $-z$ ; the reionization redshift considered is  $z = 15$ .

Figs. 14-18: The same evolution of  $x$  but, respectively, for  $z = 20, z = 30, z = 40, z = 50, z = 60$ .

Fig. 19: The evolution of the plasma temperature  $T_e$  is calculated by using an exact blackbody emission spectrum in a first order recursive calculation (see Ref. 6) and plotted as a function of  $-z$ ; the reionization redshift considered is  $z = 15$ .

Figs. 20-24: The same evolution of  $x$  but, respectively, for  $z = 20, z = 30, z = 40, z = 50, z = 60$ .